Change-Point Analysis

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Change-Point Analysis

- **Did it change?**
- **Did more than one change occur?**
- **When did the changes occur?**
- **With what confidence?**

Differences

- **Control Chart:**
  - **Update after each data point**
  - **Controls point-wise error rate**
  - **Optimal for isolated abnormal point**
- **Change-Point Analysis**
  - **Used for historical data**
  - **Controls change-wise error rate**
  - **Optimal for level shifts**

Compliment Each Other

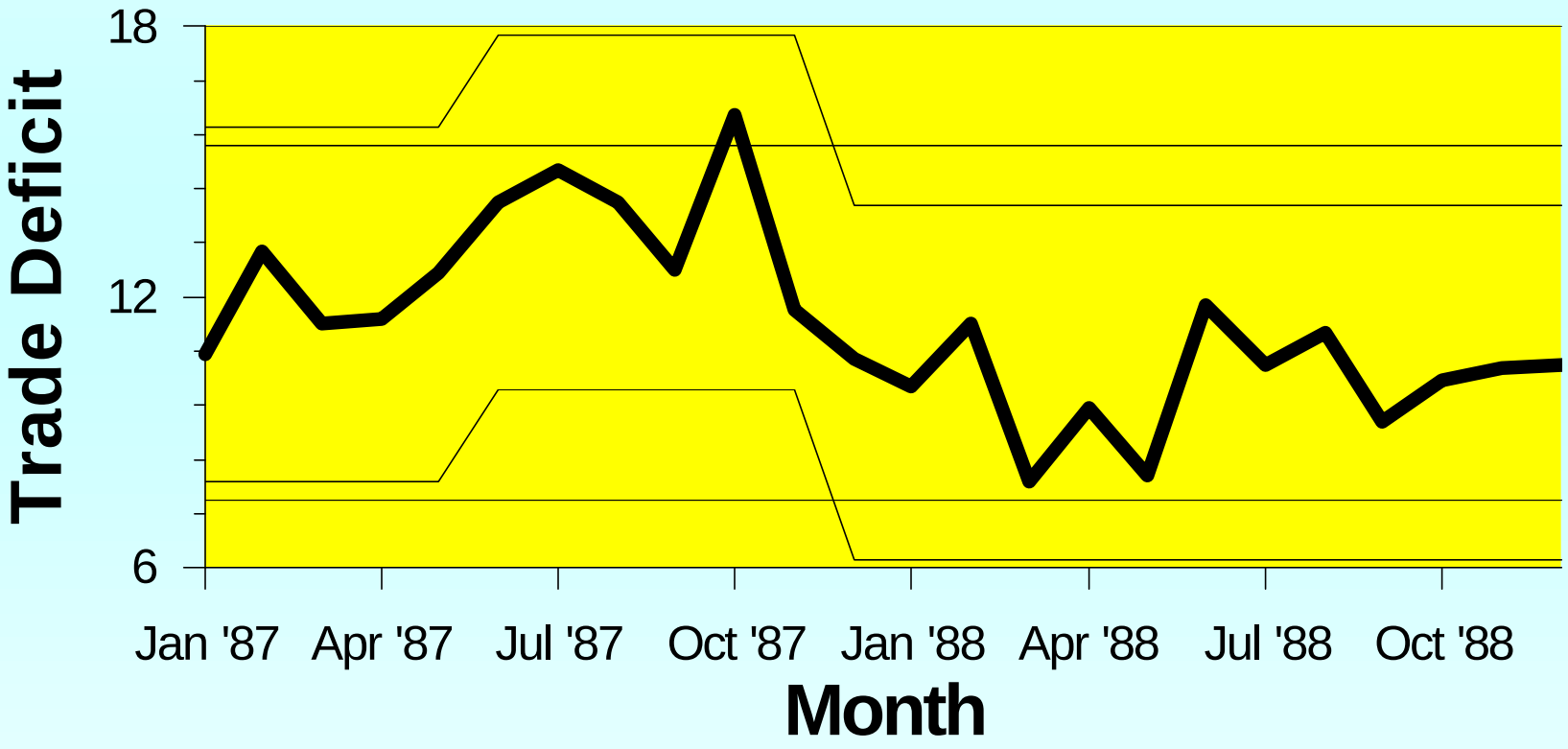
- **Use control chart on point-by-point basis to detect abnormal points and large shifts**
- **Periodically perform a change-point analysis to identify more minor shifts**

Example:

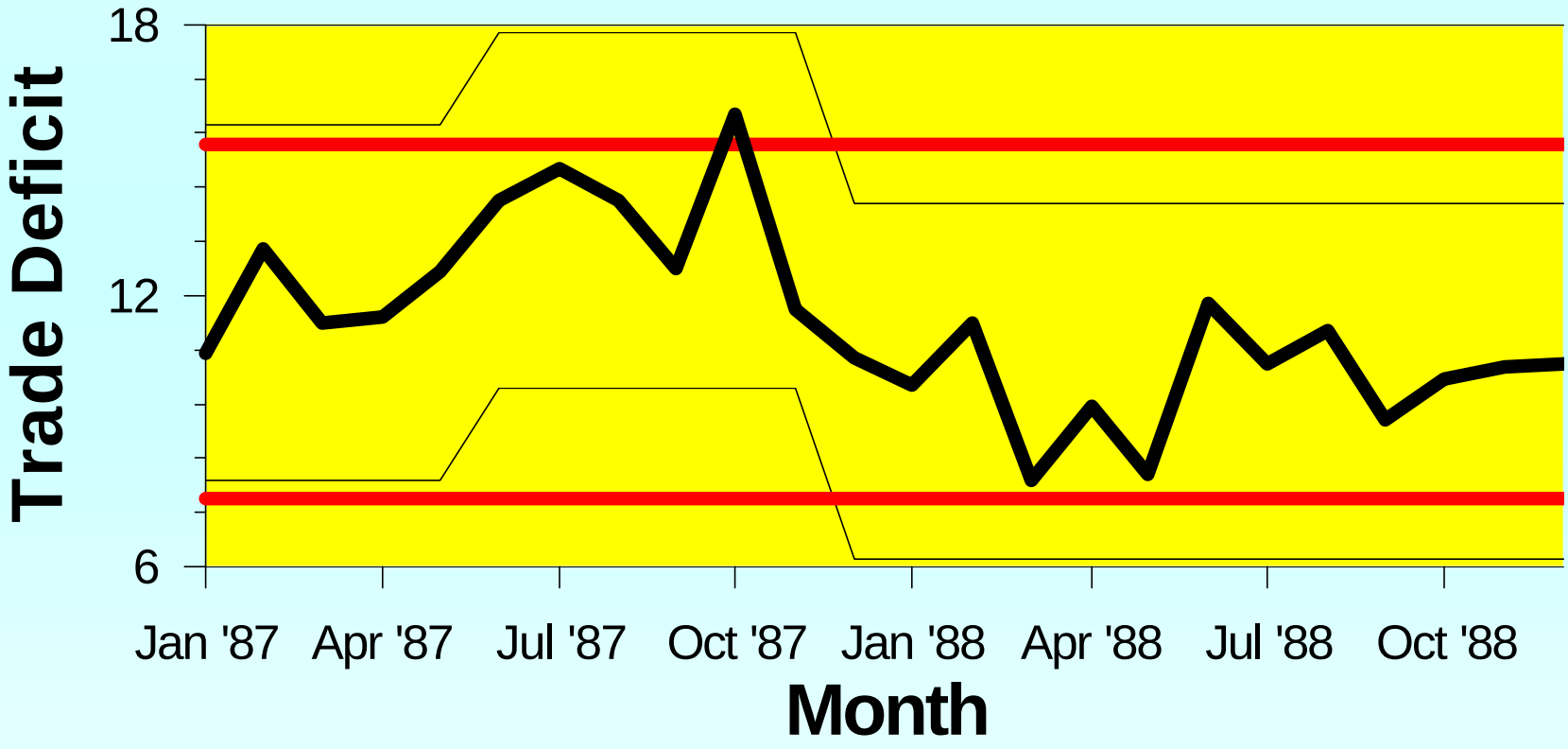
Trade Deficits 1987-1988 ($ billions)

	1987	1988
Jan	10.7	10.0
Feb	13.0	11.4
Mar	11.4	7.9
Apr	11.5	9.5
May	12.5	8.0
Jun	14.1	11.8
Jul	14.8	10.5
Aug	14.1	11.2
Sep	12.6	9.2
Oct	16.0	10.1
Nov	11.7	10.4
Dec	10.6	10.5

Plot - Trade Deficit



Individuals Chart



Individuals Chart

- **While a shift seems evident, the chart barely detects a single off-target point. (Oct ‘87)**
- **How do you interpret this out of control point?**

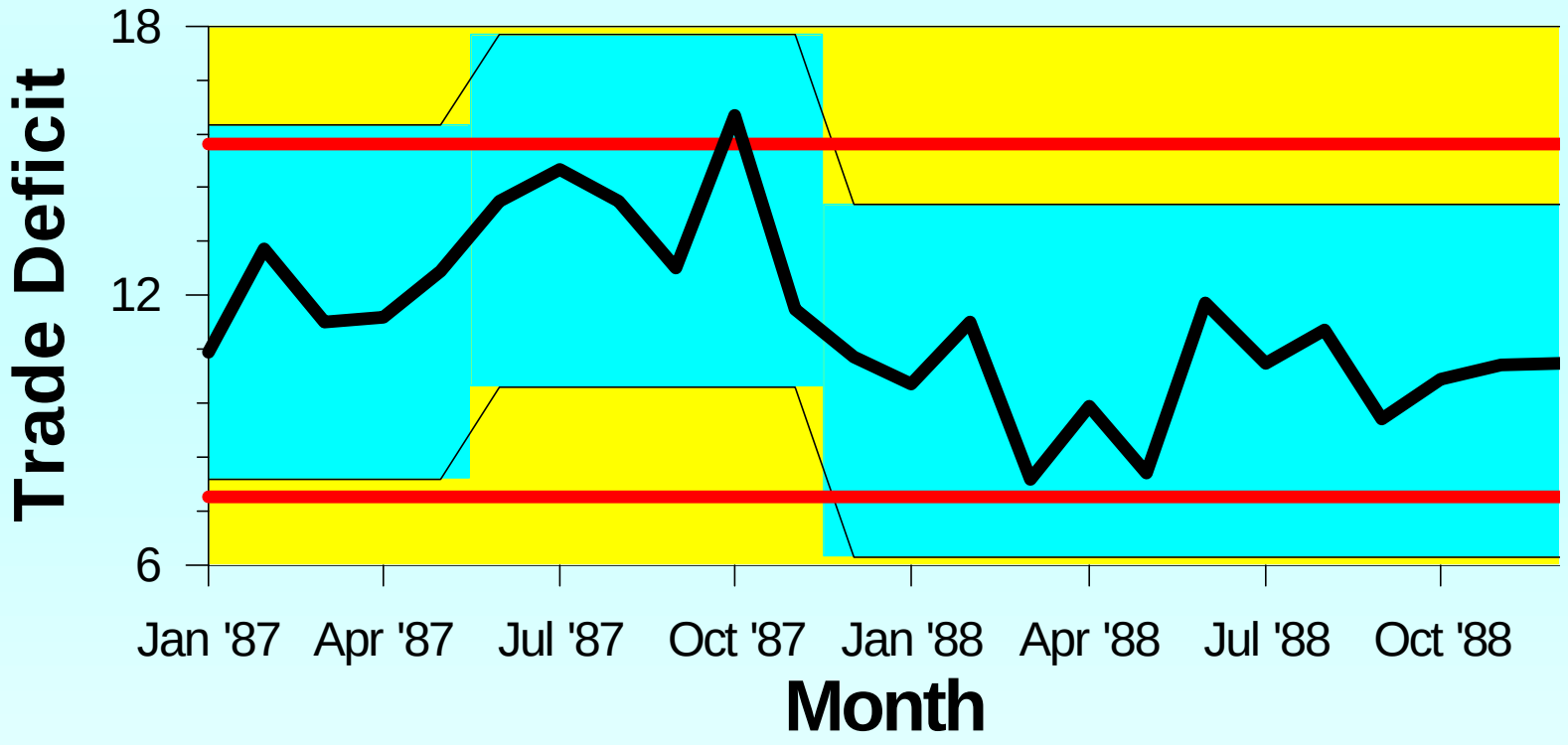
Change Point Analysis

Table of Significant Changes for Trade Deficit

Confidence Level = 90%, Confidence Interval = 95%, Bootstraps = 1000, Sampling With Replacement

Month	Confidence Interval	Conf. Level	From	To	Level
Jun '87	(May '87, Sep '87)	92.6%	11.82	13.883	2 
Dec '87	(Dec '87, Feb '88)	99.4%	13.883	10.085	1 

Plot Showing Changes



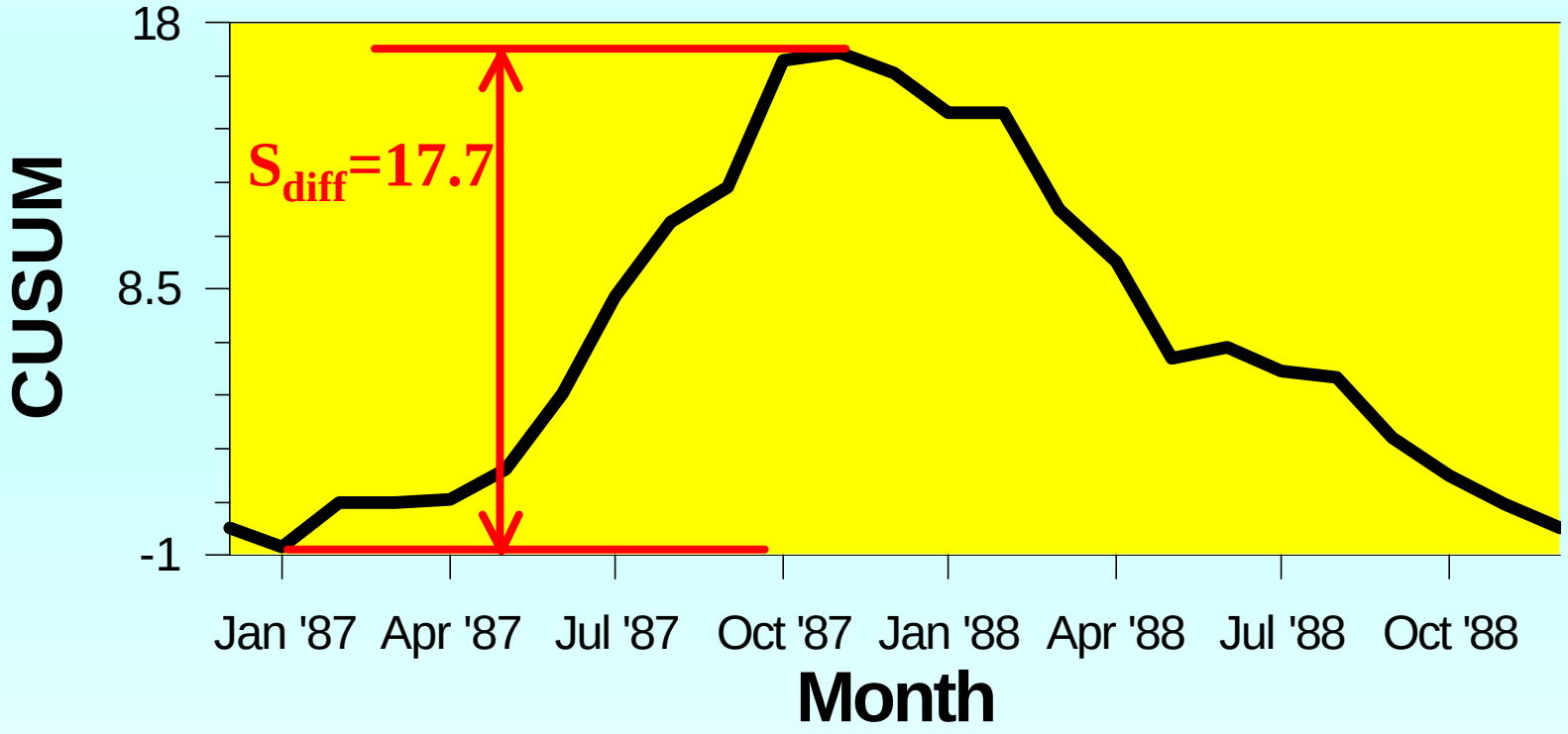
Advantages of Change Point Analysis

- Can identify multiple changes
- Better characterizes changes
- More powerful than a control chart at detecting smaller sustained changes
- Avoids false detections by controlling change-wise error rate.

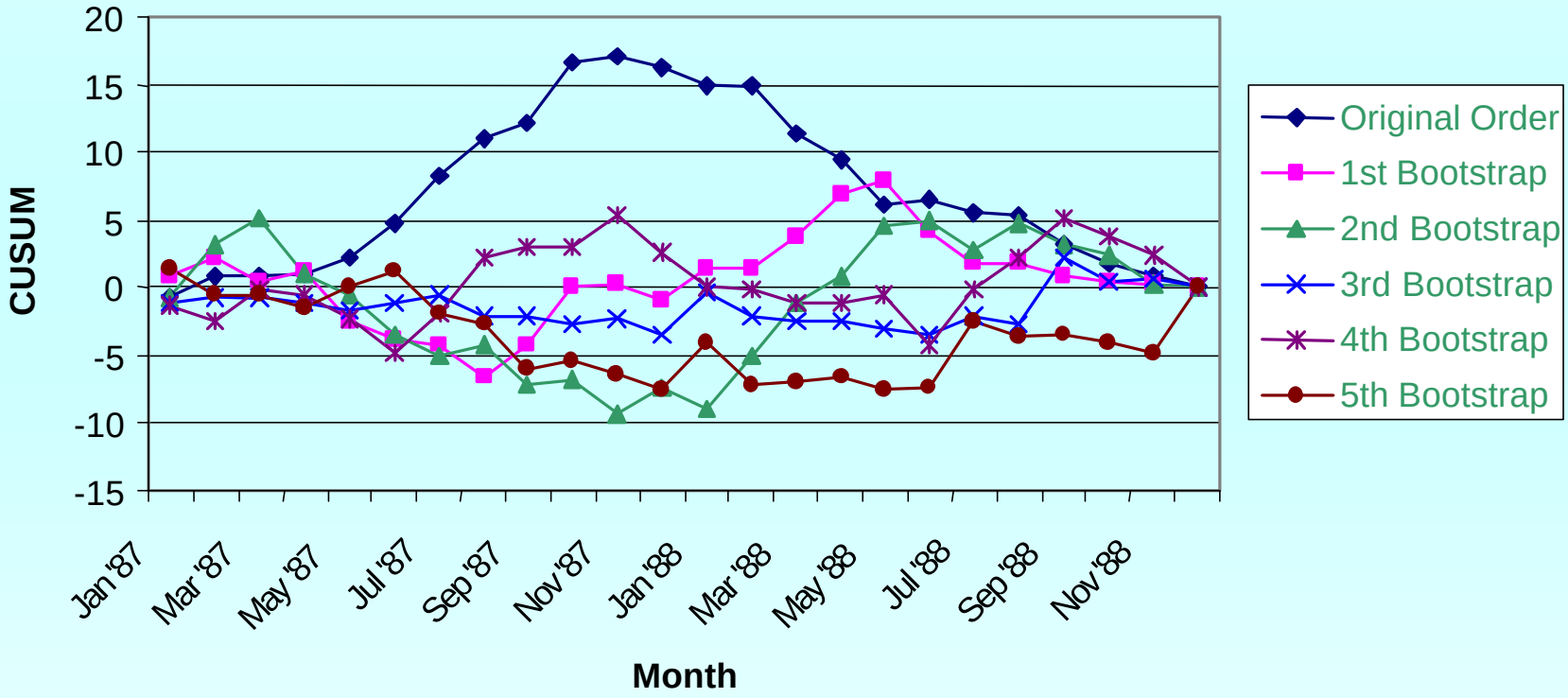
Procedure

- **Based on CUSUM chart**
- **Bootstrap analysis**
- **Iterative to detect multiple changes**

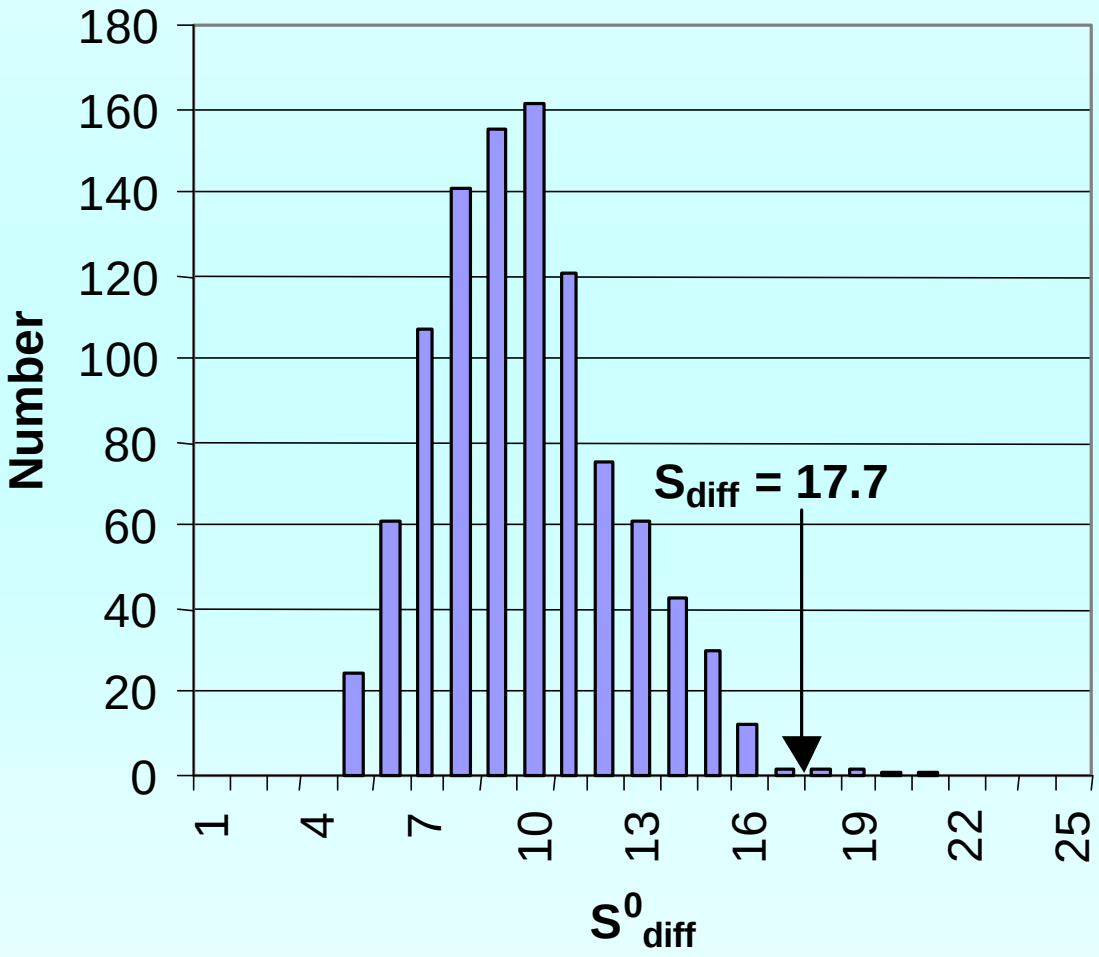
CUSUM - Trade Deficit



Bootstrap Analysis



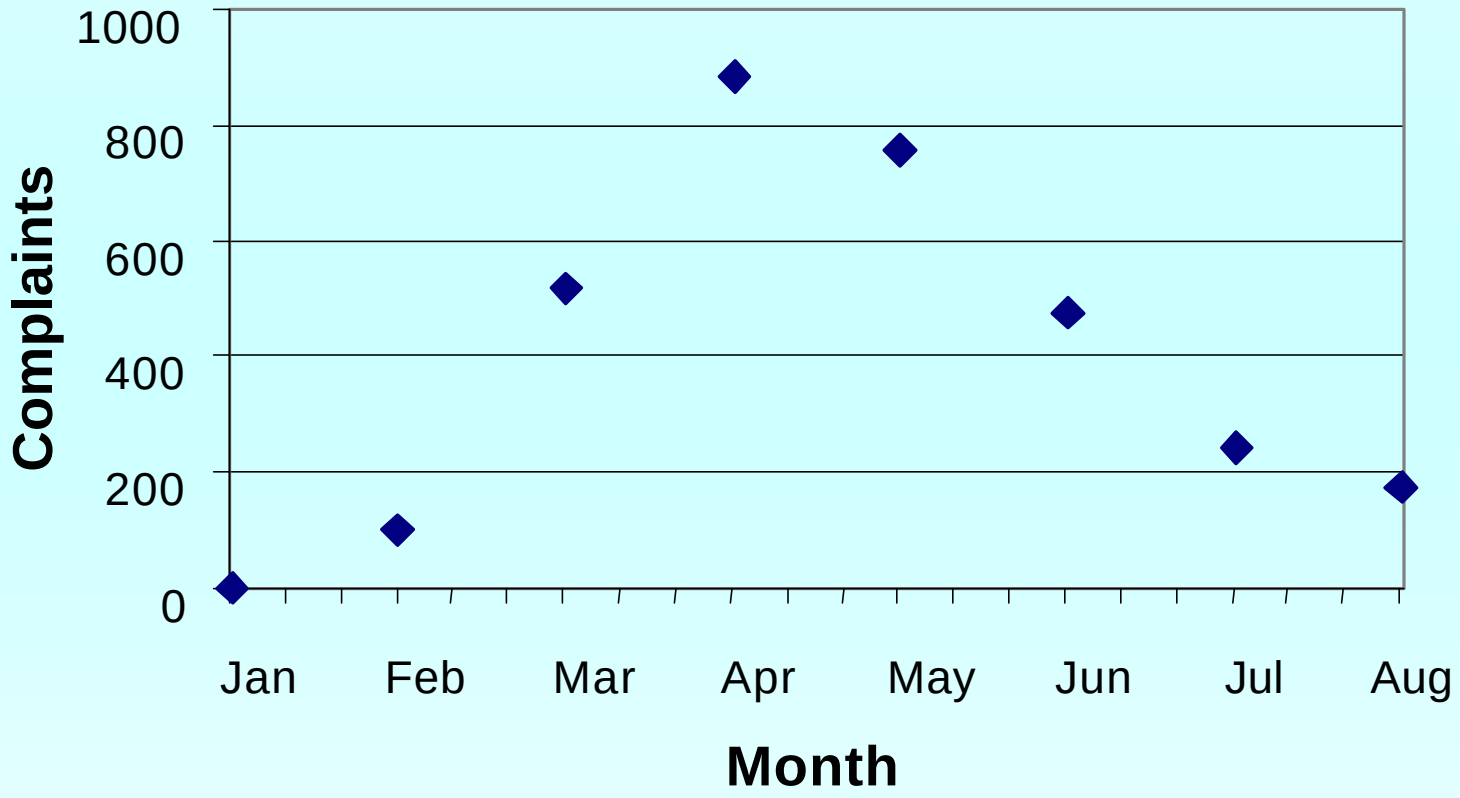
Bootstrap Analysis



Iterative Procedure

- **Once first change is detected:**
  - **Estimate time of change**
  - **Split data at this point and repeat analysis**
  - **Reestimation and point elimination procedures are also incorporated into the routine**

Complaint Data Example



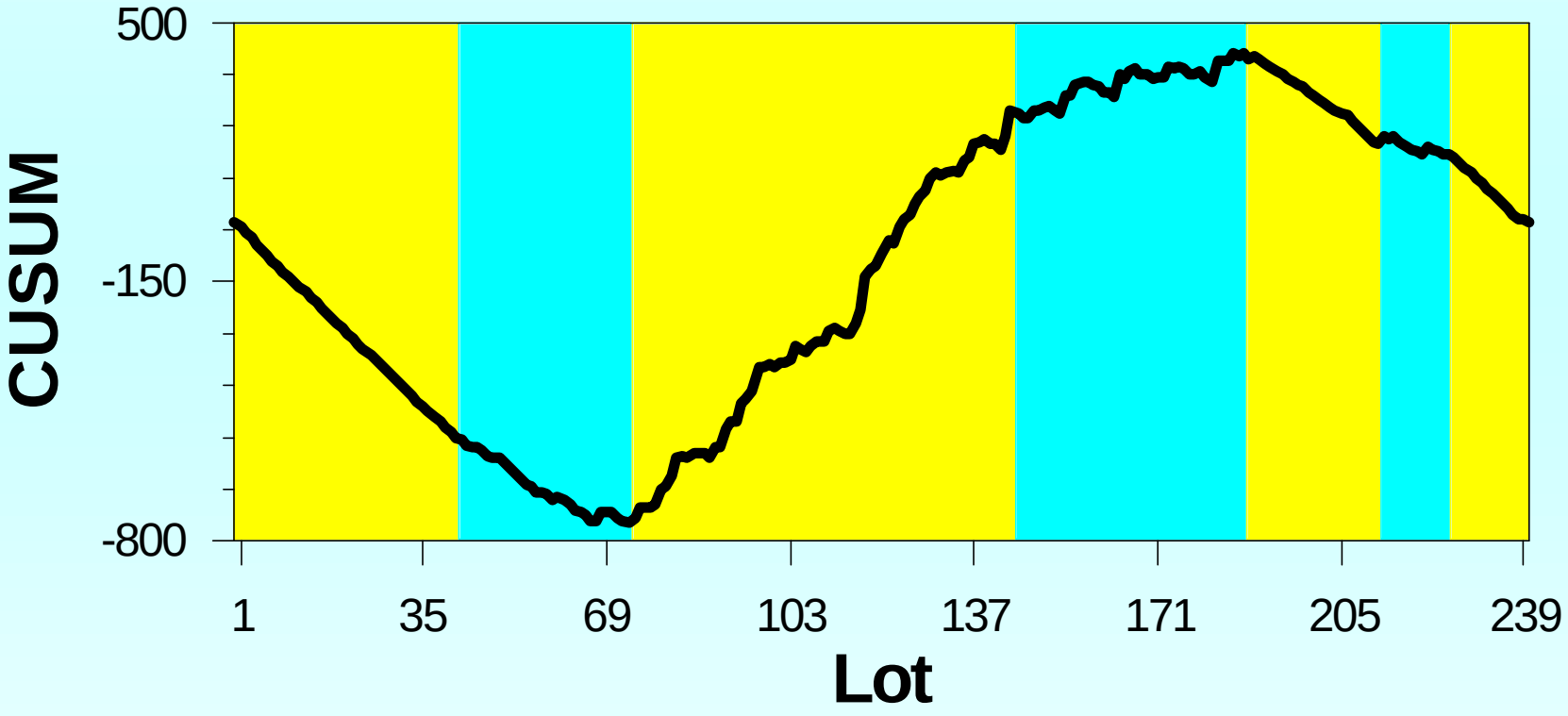
Complaints - Changes

Table of Significant Changes for Complaints

Confidence Level = 90%, Confidence Interval = 95%, Bootstraps = 1000, Sampling Without Replacement

Lot	Confidence Interval	Conf. Level	From	To	Level
42	(42, 73)	100%	0	6.5938	2
74	(72, 86)	100%	6.5938	27.662	1
145	(78, 162)	91%	27.662	16.465	2
188	(147, 188)	98%	16.465	4.24	3
213	(209, 225)	98%	4.24	11.077	5
226	(214, 228)	98%	11.077	1.6667	5

Complaints - CUSUM



Complaints - Conclusion

- Problem started around lot 42
- Problem jumped up to current level around lot 74 (within a couple of days of)
- Changes at end are due to incomplete data

More Advantages

- **Flexible – same procedure handles attribute, count and variables data**
- **Handles massive data sets with multiple changes producing hundreds of out-of-control points on a control chart**
- **Robust to outliers**

Applications

- **Problem Solving:** To pinpoint time and nature of change
- **Manufacturing:** Use whenever Shewhart chart detects out of control point to better understand change
- **Recalls:** To accurate determine fence in defensible a fashion

Applications

- **Bio and Particle Counts:** **Easily handles ill-behaved data**
- **Financial and Performance Data:** **Much more powerful than individuals chart, near-optimal against level shifts**
- **Massive and Messy Data Sets:** **Controls overall error rate, distribution free, and robust to outlyers**

So Easy Even Management Can Use It

From Excel, just highlight the data add select the change-point analysis menu item from the Tools menu. Outliers are highlighted. All assumptions are automatically checked.

- **Flexibility**
- **Simplicity**

Additional Information

- **This paper and others are posted on the website www.variation.com.**
- **Shareware package Change-Point Analyzer can be downloaded from same website.**