COMPARING THREE APPROACHES TO ROBUST DESIGN: TAGUCHI VERSUS DUAL RESPONSE VERSUS TOLERANCE ANALYSIS

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Dr. Wayne A. Taylor, Taylor Enterprises, Inc. (www.variation.com)

Robustness is a key strategy for achieving high quality - low cost products and processes. Three different approaches to robust design are commonly used: the inner/outer array approach advocated by Taguchi, the dual response approach using response surfaces and the tolerance analysis approach, which also uses response surfaces. Each of these approaches will be explained. The three approaches will then be contrasted.

The three approaches differ as to how the studies are run. Requirements for using the different approaches will be compared. In robust design, the objective is to estimate the effect that the targets of the input variables have on the variation of the output and to select the set of targets that minimize the variation while achieving the desired average. The three approaches differ as to the precision and accuracy of the resulting estimates. The accuracy and precision of the different approaches will be compared under a variety of circumstances, including study conditions that are not representative of manufacturing conditions and when some sources of variation are not included in the study.

All three approaches have strengths and weaknesses. No approach can be said to be universally superior. However, these comparisons suggest that, in most cases, the tolerance analysis approach is the best approach to use. The major weakness of the tolerance analysis approach is that it only estimates the variation caused by the inputs included in the study. It ignores other sources of variation. This weakness can be overcome using a combination of tolerance analysis and dual-response approaches.

KEY WORDS

Robust Design, Parameter Design, Dual Response, Inner/Outer Arrays, Taguchi Methods, Statistical Tolerance Analysis, Tolerance Design

INTRODUCTION

For many products and processes, the variation of the output or response variables is affected by the targets selected for the input variables. For a heat seal machine, adjusting inputs such as temperature and dwell time can affect the amount that the output seal strength varies. For a pump, adjusting inputs like stroke length and motor speed can likewise affect the amount that the output flow rate varies. When selecting targets for input variables, their effects on the outputs' variation should be considered in addition to their effects on the outputs' average. This results in what is called a robust design.

There are three common approaches to robust design: (1) the dual response approach using response surface studies, (2) the inner/outer array approach advocated by Taguchi and (3) the tolerance analysis approach also using response surface studies. Each approach is explained. There are important differences between these three approaches in terms of their requirements and the results obtained. The requirements and the results are compared to help identify when each approach should be used.

A SIMPLE EXAMPLE

The following example will be used to compare the three approaches. Suppose there is a single input variable X and a single output variable Y and that:

$$Y = -6 + 1.2\,X - 0.04\,X^2 \qquad (1)$$

A plot of the effect of X on Y is shown in Figure 1.

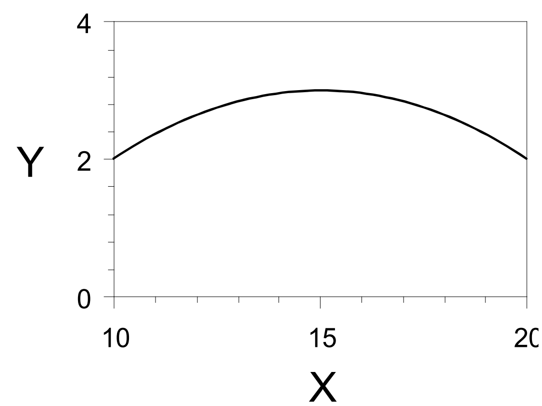

Figure 1: Effect of X on Y

Further suppose that X varies around its selected target with a standard deviation of $\sigma_X = 0.5$. We can select any target or average for X over the range of 10 to 20. We want to identify the target for X that minimizes the amount that Y varies.

Figure 2 shows why the target of X effects the variation of Y. When the target of X is 10, more variation is transmitted from X to Y due to steeper slope. This makes Y more sensitive to the variation of X. When the target is 15, the slope is less making Y less sensitive or more robust to the variation of X.

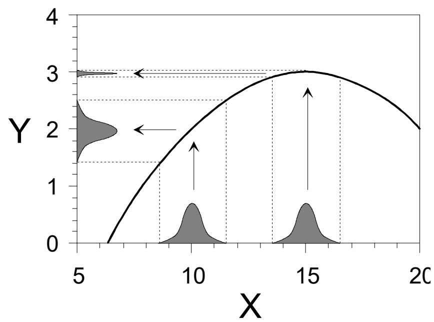

Figure 2: Achieving Robustness

EQUATION KNOWN

If the equation relating X and Y is known, an exact solution to the problem can be obtained. This is accomplished by deriving an equation for the standard deviation of Y and then finding the target of X that minimizes this equation. The equation for the standard deviation can be obtained using tolerance analysis. Details can be found in Taylor (1991), Cox (1986), Evans (1975) and many other places. These methods are also referred to as propagation of error and variation transmission analysis.

Let t_X represent the selected target or average of X. Using tolerance analysis, the following expression for the standard deviation of Y can be obtained:

$$\sigma_Y(t_X) = \sqrt{(1.2 - 0.08t_X)^2 \sigma_X^2 + 0.0032\sigma_X^4} \quad (2)$$

Figure 3 shows a plot of the standard deviation of Y. It is minimized when $t_X = 15$. The minimum standard deviation is $\sigma_Y(15.0) = 0.0141$.

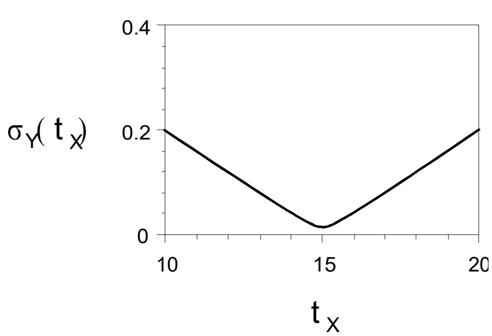

Figure 3: Plot of $\sigma_Y(t_X)$

Exact solutions or close approximations can be obtained if the equation relating Y and X is known. However, in many problems, the equation relating the inputs and outputs is not known. It must be estimated by collecting and analyzing data. Assuming the equation is unknown, we will work this problem using each of the three approaches. Once the three approaches are understood, the remainder of the article will compare the results obtained by the three methods under a variety of situations. While this problem is an over simplification of real world problems, the insight gained will prepare us for dealing with more complicated problems.

While there are many problems where the equation is unknown, there are an equal number of problems where the equation is known. Engineering and science textbooks are full of such equations. **When the equation is known, none of the 3 approaches examined in this article should be used.** Instead a tolerance analysis should be performed to obtain an exact solution. All three approaches involve approximating the actual equation with a low-order polynomial which is an unnecessary step that can result in the loss of information. See Lawson and Madrigal (1994), Taylor (1996) and Bisgaard and Ankenman (1996).

APPROACH 1: DUAL RESPONSE APPROACH

The dual response approach involves running a response surface study where both the average and standard deviation of the outputs are analyzed. The resulting equations are then used to minimize the variation while achieving a desirable average. Further information can be found in Vining and Myers (1990), and Myers, Khuri and Vining (1992).

To solve the example problem, three trials were run at equally spaced targets. Twelve observations were made per trial. Table 1 contains the data. This data was created by generating values for X according to the normal distribution and plugging these into Equation 1.

Table 1: Data for Dual Response Approach

Target	Values of Y			
10	2.34	2.03	2.02	2.21
	2.20	1.67	1.99	2.17
	2.09	1.87	2.21	1.39
15	3.00	3.00	3.00	2.97
	2.98	3.00	2.99	3.00
	3.00	2.99	3.00	3.00
20	1.72	2.19	2.04	1.93
	2.03	1.80	2.13	1.65
	1.94	1.87	1.90	2.09

Once the data was collected, the average and standard deviation for each target was calculated. The resulting values are shown in Table 2.

Table 2: Estimates of the Average and Standard Deviation of Y

Target	Average	Std. Dev.
10	2.0158333	0.2657907
15	2.9941667	0.009962
20	1.9408333	0.1645632

Quadratic polynomials were then fit to these values using regression analysis. The coefficients of an equation of the form $b_0 + b_1 t_X + b_2 t_X^2$ are estimated by:

$$\hat{\mathbf{B}} = \begin{bmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{bmatrix} = (\mathbf{X'X})^{-1}\mathbf{X'Y} \quad (3)$$

X is the design matrix shown below, where $t_1 = 10$, $t_2 = 15$ and $t_3 = 20$.

$$\mathbf{X} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 10 & 100 \\ 1 & 15 & 225 \\ 1 & 20 & 400 \end{bmatrix} \quad (4)$$

Y is the data matrix. For the standard deviation, the log of the standard deviation is generally fit instead. The resulting equation can be back-transformed to obtain an equation for the standard deviation. When fitting the log of the standard deviation, Y is:

$$\mathbf{Y} = \begin{bmatrix} \log(0.2657907) \\ \log(0.009962) \\ \log(0.1645632) \end{bmatrix} \quad (5)$$

The resulting equations for the average and standard deviation of Y are:

$$\hat{\mu}_Y(t_X) = -6.03583 + 1.2115 t_X - 0.0406333 t_X^2 \quad (6)$$

$$\hat{\sigma}_Y(t_X) = e^{(23.5082 - 3.70101 t_X + 0.121769 t_X^2)} \quad (7)$$

Most problems involve several inputs. In many cases, those inputs that have the largest effect on the variation are targeted to minimize it. The remaining inputs are then targeted to achieve the desired average. In other cases, objective functions (including signal-to-noise ratios) are optimized to obtain the best combination of the average and variation. To simplify our comparisons, we will ignore the equation for the average and assume that the objective is to simply minimize the variation. This requires finding the value of t_X that minimizes Equation 7 over the region of study. An estimate of the target minimizing the variation, denoted $\hat{t}_{min}$ can be obtained by taking the partial derivative of Equation 7 with respect to t_X, setting it to zero and then solving for t_X. This results in:

$$\hat{t}_{min} = \frac{-\hat{b}_1}{2\hat{b}_2} = 15.197 \quad (8)$$

The above equation yields the minimum so long as the result is in the region of study and b_2 is positive, i.e., the point is a minimum and not a maximum. For both these exceptions, the minimum occurs at one of the edges of the study region. At the minimum target, the predicted variation is $\hat{\sigma}_Y(\hat{t}_{min}) = 0.0099$

APPROACH 2: TAGUCHI APPROACH

The Taguchi approach differs from the dual response approach in the way the variation is estimated at the three selected targets. The Taguchi approach uses a noise (outer) array to simulate the variation of the inputs. To solve the example problem, 4 samples were collected at each of the targets: 9.5, 10.0, 10.5, 14.5, 15.0, 15.5, 19.5, 20.0 and 20.5. This keeps the total number of samples the same. Table 3 contains the data. This approach is also referred to as the inner/outer array approach. These targets represent all possible combinations of the design (inner) array **X** and noise (outer) array **N**, shown below. An estimate of σ_X is required to design the study.

$$\mathbf{X} = \begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix} \qquad \mathbf{N} = \begin{bmatrix} +\sigma_X \\ +0 \\ -\sigma_X \end{bmatrix} \quad (9)$$

Table 3: Data for Taguchi Approach

Target	Values of Y			
9.5	2.09	1.99	1.82	1.67
10.0	2.26	1.64	1.98	1.89
10.5	2.23	2.47	1.93	2.30
14.5	2.99	2.99	2.99	2.95
15.0	2.98	2.98	2.98	2.96
15.5	3.00	2.97	3.00	2.96
19.5	2.24	2.24	2.05	2.33
20.0	2.10	2.04	1.80	1.90
20.5	1.74	1.88	2.15	1.65

Once the data has been collected, the average at each target is calculated. The resulting values are shown in Table 4.

Table 4: Cell Averages

		Noise (Outer) Array		
		-0.5	+0	+0.5
Design (Inner) Array	10	1.8925	1.9425	2.2325
	15	2.9800	2.9750	2.9825
	20	2.2150	1.9600	1.8550

Estimates of the average and standard deviation of Y at each of the design array targets are obtained by calculating the average and standard deviation of the cell averages. The resulting estimates are shown in Table 5. When the design array target is 10, data is collected at 9.5, 10 and 10.5. The average of these three points is 10 and the standard deviation is $\sigma_X = 0.5$. Thus, the design array results in a systematic sample that mimics the variation of the input X. Taking the average and standard deviation of the cell averages then estimates the behavior of the output Y.

Table 5: Estimates of the Average and Standard Deviation of Y

Target	Average	Std. Dev.
10	2.0225	0.1835756
15	2.9792	0.0038189
20	2.0100	0.185135

Quadratic polynomials are then fit to these results using the same equations as before (3-5). Again, the log of the standard deviation is analyzed. The resulting equations for the average and standard deviation of Y are:

$$\hat{\mu}_Y(t_X) = -5.6686 + 1.15429 t_X - 0.038518 t_X^2 \quad (10)$$

$$\hat{\sigma}_Y(t_X) = e^{(29.3036 - 4.65023 t_X + 0.155036 t_X^2)} \quad (11)$$

The value of t_X that minimizes Equation 11 over the region of study is:

$$\hat{t}_{min} = \frac{-\hat{b}_1}{2\hat{b}_2} = 14.997 \quad (12)$$

At the minimum target, the predicted variation is $\hat{\sigma}_Y(\hat{t}_{min}) = 0.0038$. A good introduction to the Taguchi approach is Taguchi (1986).

APPROACH 3: TOLERANCE ANALYSIS APPROACH

The tolerance analysis approach starts with a response surface study on the output's average. A tolerance analysis is then performed using the predicted equation for the average. Exactly the same data is required as for the dual response approach. Everything proceeds as before up to the point that the equation for the average is obtained. The tolerance analysis approach only uses the equation for the average. The equation for the standard deviation will be ignored. Further details and examples can be found in Taylor (1991).

For the example problem, the predicted equation for the average was given in Equation 6. A tolerance analysis is then performed on this equation. This results in the following equation:

$$\hat{\sigma}_Y(t_X) = \sqrt{\begin{array}{l}(1.2115 - 0.0812666 t_X)^2 \sigma_X^2 \\ + 0.0033021 \sigma_X^4\end{array}} \quad (13)$$

An estimate of σ_X is required to use this equation to complete the analysis. This equation serves the same purpose as Equation 7 for the dual response approach and Equation 11 for the Taguchi approach. The value of t_X that minimizes Equation 13 over the study region is:

$$\hat{t}_{min} = \frac{-1.2115}{-0.0812666} = 14.908 \quad (14)$$

At the minimum target, the predicted variation is $\hat{\sigma}_Y(\hat{t}_{min}) = 0.0144$. Since the same data is required for both the dual response approach and the tolerance analysis approach, both analyses could be performed and the results compared. This might provide better results and insight than either method on its own.

COMPARISON OF RESULTS FOR EXAMPLE PROBLEM

All three approaches estimate the effect of the input's target on the output's variation (Equations 7, 11 and 13). However, they use different strategies for obtaining these estimates. The dual response approach directly observes the variation. Taguchi's approach simulates the variation using a noise array. The tolerance analysis approach predicts the variation based on an equation for the average. There are procedural differences between the three approaches. Both the Taguchi and tolerance analysis approaches require an estimate of σ_X. This estimate is used to design the Taguchi study. This limits the analysis to the value used to design the study. The tolerance analysis approach does not use the estimate of σ_X until the analysis phase. This allows alternate values to be explored as is the case when one is considering the tightening of certain tolerances. The dual response

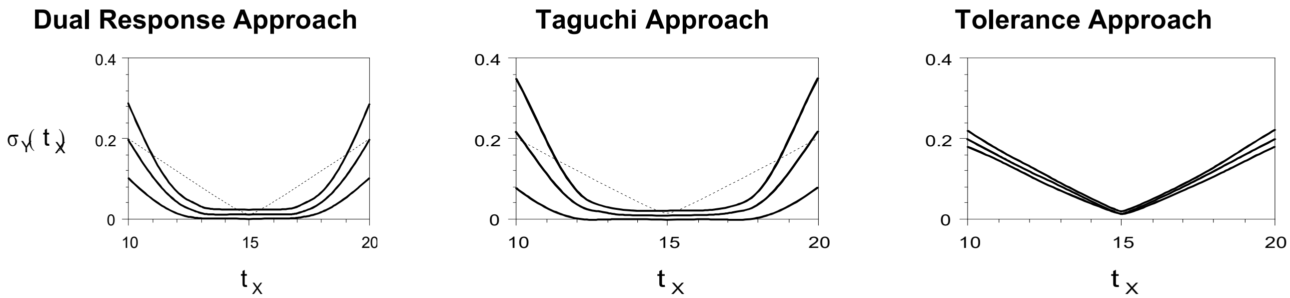

Figure 4: Estimated Standard Deviation of Y For Example Problem

approach does not require an estimate of σ_X. Instead, it directly observes the variation of Y. This requires that the variation present during the study period be representative of full-scale production. This can be difficult to ensure and requires special safeguards while the data is being collected. Another procedural difference is that the Taguchi approach requires a larger number of adjustments and finer adjustments which may be difficult to make and which increase the cost of the study.

Besides such procedural differences, their are also differences in the accuracy and precision of the resulting estimates. To compare the resulting estimates, 100,000 sets of data were generated and analyzed using each approach. Figure 4 shows the results of estimating $\hat{\sigma}_Y(t)$. The center line represents the average of $\hat{\sigma}_Y(t)$ and the band represents ±2 standard deviations. The true value is also shown as a dotted line. The tolerance analysis approach has the narrowest band around the true value indicating that it provides the greatest accuracy and precision. Why the difference in curve shapes? Equation 1, the true equation for Y, is a quadratic polynomial. However, Equation 2, the true equation for the standard deviation, is not a quadratic polynomial nor can it be closely approximated by one. The dual response approach and Taguchi approach attempt to fit a quadratic polynomial to the standard deviation. As a result, there is significant lack of fit in between the three fitted points. The tolerance analysis approach fits a quadratic polynomial to the average which results in an accurate fit. This example illustrates that equations for standard deviations tend to be more complex than equations for the average. In general, the order of the polynomial required to fit the standard deviation is double that for the average. Thus if a 2nd-order polynomial fits the average well, a 4th-order polynomial may be required for the standard deviation. This is a result of the fact that the standard deviation of Y depends on the $E\{Y^2\}$. All three methods, will be effected by lack of fit due to incorrect models. However, the tolerance analysis approach will generally be effected less since it fits the average instead of the variation.

For each of the three approaches, Table 6 compares the estimates of both t_{min} and the variation at t_{min}. All values are reported to the number of digits estimated to be accurate. The true values are shown at the top of the table. All three approaches provide an accurate (unbiased) estimate of t_{min}. However, the tolerance analysis approach is more precise (smaller standard deviation) at estimating t_{min} than the other two approaches. The dual response approach is second. Both the dual response approach and Taguchi approach underestimate the variation at t_{min}. The tolerance analysis approach provides an accurate estimate that is ten times more precise than the other two approaches.

Table 6: Estimates of Minimum Target and Standard Deviation at Minimum Target For Example Problem

Approach	$t_{min} = 15$		$\sigma_Y(t_{min}) = 0.0141$	
	Average	Std. Dev.	Average	Std. Dev.
Dual Response	15.00	0.17	0.0127	0.0062
Taguchi	15.00	0.26	0.0104	0.0063
Tolerance Analysis	15.00	0.112	0.0141	0.0006

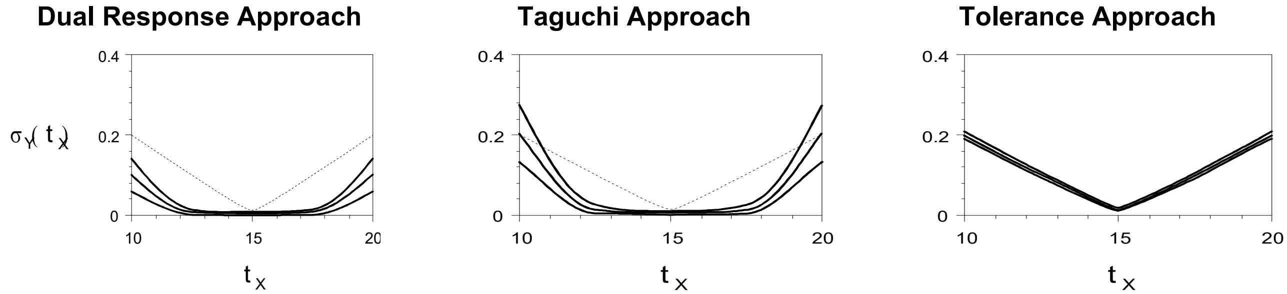

Figure 5: Estimated Standard Deviation of Y For Three Approaches When Study Conditions Are Not Representative of Manufacturing

STUDY CONDITIONS NOT REPRESENTATIVE OF MANUFACTURING

In the example problem, it was assumed that the variation of X during the study was representative of its variation during actual manufacturing. This is frequently not the case. It is not uncommon for the variation during the study to be less than what will be experienced under more extended periods of production. Frequently, limited material is available, the process is better controlled, and components of variation, such as roll-to-roll variation and tank-to-tank variation, are missed. To determine what effect running the study under conditions not representative of manufacturing has on the results, 100,000 new datasets were generated and analyzed. A value of $\sigma_X = 0.25$ was used to generate data representing study conditions. It is still assumed that $\sigma_X = 0.5$ under long-term manufacturing so that the correct answers are the same as before. Further, a value of $\sigma_X = 0.5$ was still used to design the Taguchi study and to perform the tolerance analysis.

The results are shown in Figure 5 and Table 7. Not too surprisingly, the dual response approach ends up badly underestimating the variation of Y across the entire curve. However, the ability to estimate t_{min} does not suffer. In fact, it improves slightly. This might not be the case if other inputs were also included in the study. Suppose a study was run where the input contributing the most variation only varied over a narrow range while a less important input varied over its full range. The dual response approach could end up making the process robust to the second input at the expense of making it more sensitive to the first. While the selected targets might optimize the process for the study conditions, they might perform poorly in manufacturing. **When using the dual response approach, study conditions should be as representative of long term manufacturing as possible.** This can significantly add to the cost of the study.

Both the Taguchi and tolerance analysis approaches benefit from the reduced variation during the study. In both case the standard deviation of the t_{min} estimate is reduced by at least 50% compared to Table 6. **When using the Taguchi and tolerance analysis approaches, variation during the study should be kept to a minimum.** The tolerance analysis approach maintains its advantage over the other two approaches with respect to precision of the t_{min} estimate. Both the dual response approach and Taguchi approach underestimate the variation at t_{min}. Again the tolerance analysis approach provides an unbiased estimate that is ten times more precise than the other two approaches.

Table 7: Estimates of Minimum Target and Standard Deviation at Minimum Target When Study Conditions Are Not Representative of Manufacturing

Approach	$t_{min} = 15$		$\sigma_Y(t_{min}) = 0.0141$	
	Average	Std. Dev.	Average	Std. Dev.
Dual Response	15.00	0.118	0.0032	0.0015
Taguchi	15.00	0.096	0.0068	0.0027
Tolerance Analysis	15.00	0.051	0.0141	0.0003

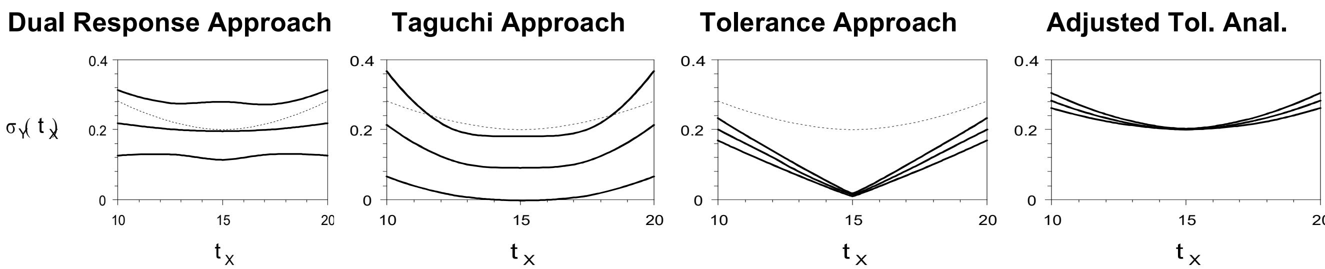

Figure 7: Estimated Standard Deviation of Y When Measurement and Other Variation

MEASUREMENT ERROR AND OTHER VARIATION

In the example problem, all variation is due to X. There are no other sources of variation, including measurement. In this section, we will add additional variation to our model and repeat the comparison. Suppose we add to Equation 1 a second input E representing the other sources of variation. The new model becomes:

$$Y = -6 + 1.2\,X - 0.04\,X^2 + E \qquad (15)$$

Assume E has average $\mu_E = 0$ and standard deviation $\sigma_E = 0.2$. Then the true formula for the standard deviation changes to:

$$\sigma_Y(t_X) = \sqrt{(1.2 - 0.08t_X)^2\sigma_X^2 + 0.0032\sigma_X^4 + \sigma_E^2} \qquad (16)$$

Figure 6 shows a plot of the standard deviation of Y. It is still minimized when $t_X = 15$. The resulting minimum standard deviation changes to 0.2005.

To compare the resulting estimates, another 100,000 sets of data were generated and analyzed using each approach. As in the previous section, a value of $\sigma_X = 0.25$ was used to generate data. The precision and accuracy of the estimated variation of Y are shown in Figure 7. The tolerance analysis approach has the narrowest band, indicating that it provides the greatest precision. However, this approach badly underestimates

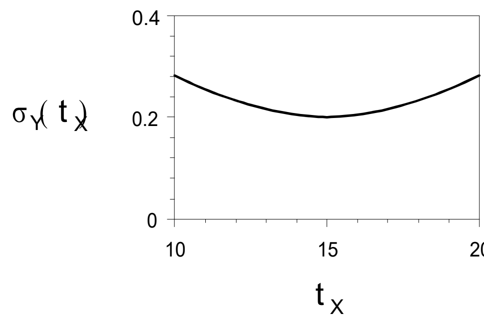

Figure 6: Plot of $\sigma_Y(t_X)$ When Other Variation

Y's variation. This is because it only estimates the variation resulting from X. It ignores variation from other sources. The Taguchi approach also underestimates Y's variation for the same reason, although not nearly as badly. The dual response approach continues to underestimate the variation due to study conditions not representative of manufacturing.

The reason the tolerance analysis approach underestimates Y's variation is that it ignores the measurement variation and the variation resulting from variables not included in the study. This can be fixed by being sure to include all possible sources of variation in the study. It may also be necessary to adjust the estimated variation to account for the measurement variation. Suppose $\hat{\sigma}_E$ is an estimate of the measurement variation. One could then estimates Y's variation as follows:

Table 8: Estimates of Minimum Target and Standard Deviation at Minimum Target When Measurement and Other Variation

Approach	$t_{min} = 15$		$\sigma_Y(t_{min}) = 0.2005$	
	Average	Std. Dev.	Average	Std. Dev.
Dual Response	15.00	3.54	0.170	0.0313
Taguchi	15.00	1.82	0.0820	0.0413
Tolerance Analysis	15.00	0.115	0.0141	0.0010
Adjusted Tol. Anal.	15.00	0.115	0.2005	0.0000

$$\hat{\sigma}^2_{Y-Adjusted}(t_X) = \sqrt{\hat{\sigma}^2_{Y-Original}(t_X) + \hat{\sigma}^2_E} \quad (17)$$

Assuming $\hat{\sigma}_E = 0.2$, the adjusted results are also shown in Figure 7. This results in a highly accurate and precise estimate of Y's variation. A similar adjustment can be made for the Taguchi approach.

Table 8 shows the average and standard deviation of the resulting estimates of t_{min}. The standard deviation of all three methods increased from the previous section (Table 7). The presence of additional variation only makes it more difficult to estimate t_{min}. However, the standard deviation of the tolerance analysis approach only increases 30% while the other two approaches increase 2 to 3 fold. With variation from other sources present, the tolerance analysis approach has a standard deviation of 1/15 that of the Taguchi approach and 1/30 that of the dual response approach. This confirms what the author has experienced in practice, that the tolerance analysis approach is able to repeatedly discover effects missed by the other two approaches.

Table 8 also indicates that all three methods underestimate the minimum standard deviation. The tolerance analysis approach is worse. However, by adjusting for measurement error and being sure to include all possible sources of variation, the tolerance analysis approach gives amazingly accurate and precise estimates.

SUMMARY

All three approaches to robustness have been highly successful. Numerous case studies attest to this fact. Any method of using designed experiments and addressing robustness is better than none. At issue is whether one approach offers significantly greater benefits than the others and can be said to be the best demonstrated practice.

Robustness applies to a variety of products and processes including electronic hardware, plastics manufacturing, chemical processes and metal processing. Such applications vary greatly in the cost of collecting data, type of objectives, and so on. There may not be a single method that is best for all these applications. What has worked well in one industry may not be the best approach in another. This article attempts to compare the three approaches under a variety of circumstances to help identify differences in the performance between the three approaches. In addition, there are procedural differences that must be considered. These differences are summarized in Table 9.

The tolerance analysis approach is frequently the best approach. It far outperforms the other approaches in terms of accuracy and precision of its estimates. It is also is the least expensive approach. Further, it has the enormous advantage that the variation or tolerances for the input do not have to be specified until the analysis phase, allowing alternate designs or conditions to be analyzed as well. Robustness alone might not be enough to achieve your objectives. Tightening of some tolerances may be necessary. The tolerance analysis approach allows alternate tolerances to be explored and optimized without requiring further collection of data.

The single weakness of the tolerance analysis approach is that it requires that all important sources of variation be included in the study. Special care should be taken to consider all variables that might affect the product or process. This generally requires running screening experiments or fractional factorial designs first on a large number of inputs and then augmenting it into a response surface study on the key inputs.

If one is studying a small number of the most important inputs, the dual response approach should be used. It is the only approach that achieves robustness against sources of variation not formally included in the study. Because of the poor precision of this approach, it would be best to take 50 to 100 samples per trial. It should be routine practice to analyze the standard deviation anytime the average is analyzed. While the dual response approach may miss some inputs due to its poor precision, inputs that it finds can lead to significant improvements without the need for additional data.

Since the dual response approach and tolerance analysis approach can use the same data, the two approaches can be used together. Such studies should be run under conditions as representative of manufacturing as possible. While this is not the ideal conditions for the tolerance analysis approach, the tolerance analysis approach still has good precision. This allows the predicted performance from the tolerance analysis approach to be compared to the observed variation from the dual response approach and serves as a check and balance.

Finally, no method is foolproof. None of the three methods works when the major source of variation has not yet been identified as an input and thus is not included in the study and this input does not vary much during the period the study is performed. An example might be a seasonal variable not considered.

Table 9: Comparison of Three Approaches to Robust Design

	Dual Response	Taguchi	Tolerance Analysis
Method of Estimating Variation	Direct Observation	Uses noise array to simulate variation of inputs	Predicts variation using equation for average
Procedural Requirements	Requires study conditions to be representative of manufacturing Requires multiple observations per trial	Works best when the variation present during the study is minimized Requires numerous fine adjustments to be made Requires estimates of variation of inputs to design study Can be run with one observation per cell	Works best when the variation present during the study is minimized Requires estimates of variation of inputs to analyze study Can be run with 1 observation per trial
Cost of Running Experiment	Cost are increased due to extra precautions to insure study conditions are representative of manufacturing	Costs are increased due to need to make numerous fine adjustments	Lowest cost approach
Estimated Variation	Under estimates variation if study conditions are not representative of manufacturing. Least precise method	Under estimates variation if important sources of variation are not included in the study Low precision but better than dual response approach	Under estimates variation if important sources of variation are not included in the study Including all sources of variation and possible adjusting for measurement error results in accurate predictions of the variation High precision
Target Minimizing Variation	If study conditions are not representative of manufacturing, will optimize for study conditions and may fail in manufacturing Least precise method. However this lack of precision can be partially overcome by taking larger number of samples per trial. Sample sizes of 50-100 are more appropriate	If an important source of variation is not included, will optimize only for sources of variation included in the study and may fail in manufacturing Low precision but better than the dual response approach	If an important source of variation is not included, will optimize only for sources of variation included in the study and may fail in manufacturing High precision. Standard deviation 1/15 that of the Taguchi approach and 1/30 that of the dual response approach
Other	Only method that can achieve robustness against unidentified source of variation		Only method that can explore alternate tolerances for inputs without requiring additional data

Most debates about Taguchi Methods center around the dual response approach versus Taguchi. While care should be taken in concluding too much from these limited examples, the results presented indicate that a third lesser known approach called the tolerance analysis approach is in many if not most cases the best approach to use. Even better results can be obtained by combining the dual response and tolerance analysis approaches. Finally, all three of these approaches are only appropriate when the equation relating the inputs and the output are unknown. If the equation is known, none of these approaches should be used. Instead exact solutions can be obtained.

REFERENCES

Bisgaard, Soren and Ankenman, Bruce (1996), "Analytic Parameter Design", Quality Engineering, Vol. 8, No. 1, pp. 175-96.

Cox, Neil D. (1986), *How to Perform Statistical Tolerance Analysis*, ASQC, Milwaukee, WI.

Evans, David H. (1975), "Statistical Tolerancing: The State of the Art," Journal of Quality Technology, Vol. 7, No. 1, pp. 1-12.

Lawson, John S. and Madrigal, J. L. (1994), "Robust Design Through Optimization Techniques", Quality Engineering, Vol. 6, No. 4, pp. 593-608.

Taguchi, Genichi (1986), *Introduction to Quality Engineering - Designing Quality into Products and Processes*, Asian Productivity Organization, Tokyo.

Taylor, Wayne A. (1991), *Optimization and Variation Reduction in Quality*, Taylor Enterprises Inc, Libertyville, IL.

Taylor, Wayne A. (1996), *VarTran*, Taylor Enterprises Inc, Libertyville, IL.